

Course Title: Advanced Mathematics Foundations for Engineers

Instructor: Dr. Arta Jamshidi, ECE faculty, University of Tehran.

Description

We introduce a new course that provides an in-depth mathematical background for engineering students with emphasis on real applications. The course is intended for graduate students who desire to build a solid mathematical thinking to pursue their applied or analytical research career. This course highlights topics in mathematical analysis, algebra, topology and differential geometry. There will be emphasis on practical applications of various mathematical concepts. The course aims at motivating students to find relevance of what they learn to scientific literature and bring insights from their respected field of study. The course is designed for control graduate students; however participation is welcome from all engineering and science majors.

Syllabus

- 1) Point set Topology
 - Set theory and Logic
 - Topological Spaces and Continuous Functions
 - Compactness and Connectedness

- 2) Abstract and Linear Algebra
 - Groups, Rings, Fields
 - Vector Spaces
 - Subspaces and Quotient Spaces
 - Duality and Dual Spaces

- 3) Real Analysis and Functional Analysis
 - Sequences and Series
 - Continuity and Differentiation
 - Riemann and Lebesgue Integration
 - Sequences and Series of Functions
 - Banach and Hilbert Spaces
 - Projection Theorem and Best Approximation
 - Hahn–Banach Theorem

- 4) Intro to Differential Geometry with Applications
 - Manifolds
 - Differential Structures, Charts
 - Morphism
 - Vector Bundles
 - Tensors and Differential Forms

Evaluation and Grading Policy

The course is evaluated based on active class participation, in terms of providing new ideas and insights that relate to topics of each class session. There will be direct homework assignments for each topic (a few problems that students select). The assignments could be done in collaboration, but written up individually. Research projects in scientific literature that relate the discussed topics in the class to students' respected field of study would form the major part of the evaluation. There would be possibilities for students to follow their interests through various projects.

References:

- 1) *From Vector Spaces to Function Spaces Introduction to Functional Analysis with Applications*, Y. Yamamoto, SIAM Press, 2012.
- 2) *Topology A First Course*, J. Munkres, Prentice Hall, NJ, 2000.
- 3) *Abstract Algebra*, D. Dummit and R. Foote, Wiley Press.
- 4) *Linear Systems*, T. Kailath, Prentice Hall, 1980.
- 5) *Matrix Mathematics: Theory, Facts, and Formulas*, 2nd ed., D. S. Bernstein, Princeton Univ. Press, 2009.
- 6) *Convex Optimization*, S. Boyd and L. Vandenberghe, Cambridge Univ. Press, 2004.
- 7) *Functional Analysis*, W. Rudin, McGraw–Hill, New York, 1973
- 8) *Real and Complex Analysis*, W. Rudin, McGraw–Hill, New York, 1966.
- 9) *Geometric Control Theory*, V. Jurdjevic, Cambridge University Press, 1997.
- 10) *An Introduction to Differentiable Manifold and Riemannian Geometry*, W. M. Boothby, Academic Press, London, U.K.
- 11) *Calculus on manifolds - Vol. 1*, M. Spivak, W. A. Benjamin, Inc., New York, 1965.

Short Biography:

Dr. Jamshidi is an expert in data analysis. He earned his Ph.D. in Mathematics in 2008 developing novel nonlinear signal processing and function approximation tools that serve various applications. He holds an M.Sc. in mathematics, an M.Sc. and B.Sc. (Summa Cum Laude) in Electrical Engineering. He has undertaken interdisciplinary post-doctoral research programs at Imperial College London and Princeton University working on imaging systems, modelling multi-scale dynamics and renewable energy systems. His research interests include big data analysis, signal and image processing, optimization and mathematical modeling. He has published in SIAM and IEEE Journals and holds two US patents.